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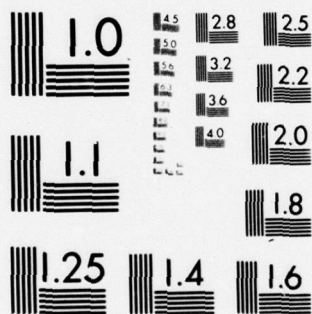
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SYSTEMS ANALYSIS DIRECTORATE  
ACTIVITIES SUMMARY  
NOVEMBER 1976

DECEMBER 1976

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20. ABSTRACT (Continue on reverse side if necessary and identify by block number) This monthly publication contains Memoranda for Record and other technical information that summarize the activities of the Systems Analysis Directorate, US Army Armament Command, Rock Island, IL. The subjects dealt with are: Protective Mask, XM29; Footprints of Army and Navy Guided Projectiles; Copperhead and HELLFIRE.		

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## Section I. GENERAL

1. This monthly publication summarizes the activities of the Systems Analysis Directorate. The purpose of this note is to give wider and more timely distribution on subjects of concern to the command.
2. The most significant Memoranda for Record (MFR's) and other technical information will be published as notes or reports at a later date.
3. In order to assure accurate distribution of this publication, addition or deletion of addresses to/from the DISTRIBUTION LIST are invited and should be forwarded to the address below.
4. Inquiries applicable to specific items of interest may be forwarded to Commander, US Army Armament Command, ATTN: DRSAR-SA, Rock Island, IL 61201 (AUTOVON 793-4483/4628).

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**Section II. MEMORANDA AND OTHER TECHNICAL INFORMATION**

**Memoranda for Record and other technical information are grouped according to subject, where applicable, and in chronological order.**

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ADDENDA TO COMPARISON OF FOOTPRINTS  
OF ARMY AND NAVY GUIDED PROJECTILES

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DRSAR-SAM

11 NOV

MEMORANDUM FOR RECORD

SUBJECT: Addenda to Comparison of Footprints of Army and Navy Guided  
Projectiles

1. References:

a. Memorandum for Record, DRSAR-SAM, 16 Jul 76, subject: Computer Simulation Study of Navy 5"/155mm Sleeved Round for Guidance Accuracy and Footprint.

b. Memorandum for Record, DRSAR-SAM, 31 Aug 76, subject: Computer Simulation Study of Copperhead (CLGP) for Guidance Accuracy and Footprint.

c. Briefing for DDR&E by George Schlenker and Richard Heider, 14 Sep 76, subject: Inputs to Army-Navy Guided Projectile Commonality Study.

2. Introduction.

At the referenced briefing (Ref 1c), the undersigned presented inputs to and results from the studies of Refs 1a and 1b to an audience of Army, Navy, and DOD personnel.

The Navy personnel present took issue with the value of the seeker field of view (FOV) used by DRSAR-SA for the Navy round. Discussion established that the value furnished SA in the Navy's system specification of March 1976 had been superseded in May 1976 without SA having been notified.

In order to assure that the performance estimates of the Navy projectile were not unfairly pessimistic, the undersigned agreed to repeat the experiments on the Navy footprint, using the newer value of  $\pm 17^\circ$  instead

DRSAR-SAM

MEMORANDUM FOR RECORD

SUBJECT: Addenda to Comparison of Footprints of Army and Navy Guided  
Projectiles

of the older  $\pm 12^\circ$  for the FOV.

3. Experimental Results.

The footprint experiments were performed in the manner described in previous studies (see Ref 1a), but with a larger number of experimental points to better define the footprints.

Results of the present experiment, including, for the first time, ceilings above 3000 ft, are displayed in Figures 1-3 and Table 1.

4. Comments

a. Comparison of the present results with the previous ones (Ref 1a) reveals the following:

(1) At 6 km gun-to-target range (GTR), the sizes of the footprints increase significantly. However, the utility of the footprints is essentially unchanged. See comments on these footprints in Refs 1a and 1b.

(2) At 12 km GTR the increases in size result in increased utility.

(3) At 18 km GTR the differences in sizes are entirely attributable to the increased precision of the present experiment. No increase is attributable to the increased FOV. Indeed, the output of the computer simulation reveals that, for 18 km GTR and ceiling  $\leq 3000$  ft, the boundary of the footprint is everywhere determined by maneuverability, not ability to see and acquire the target.

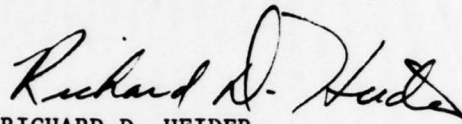
b. Substitution of the new values for the old does not change the

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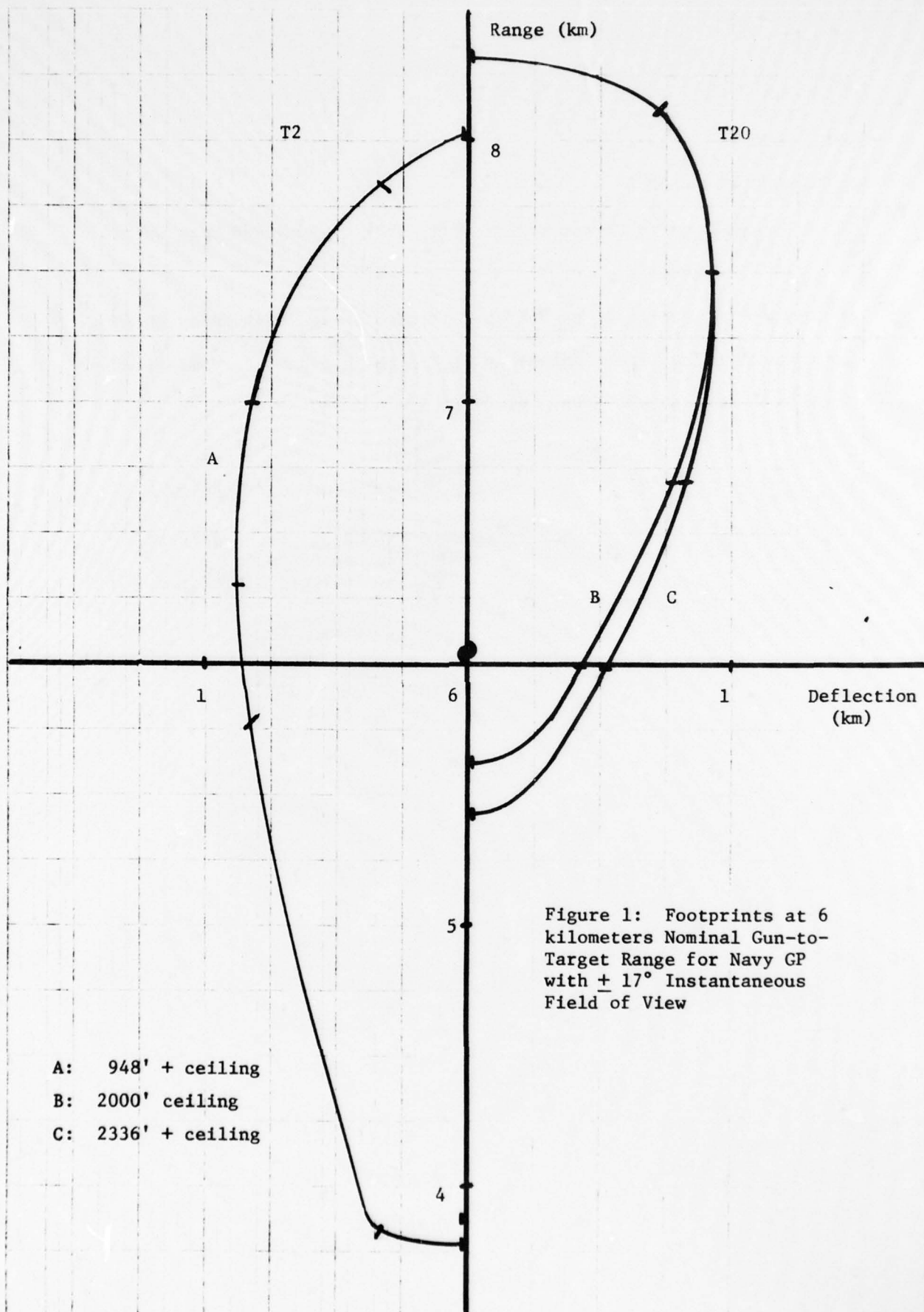
MEMORANDUM FOR RECORD

SUBJECT: Addenda to Comparison of Footprints of Army and Navy Guided  
Projectiles

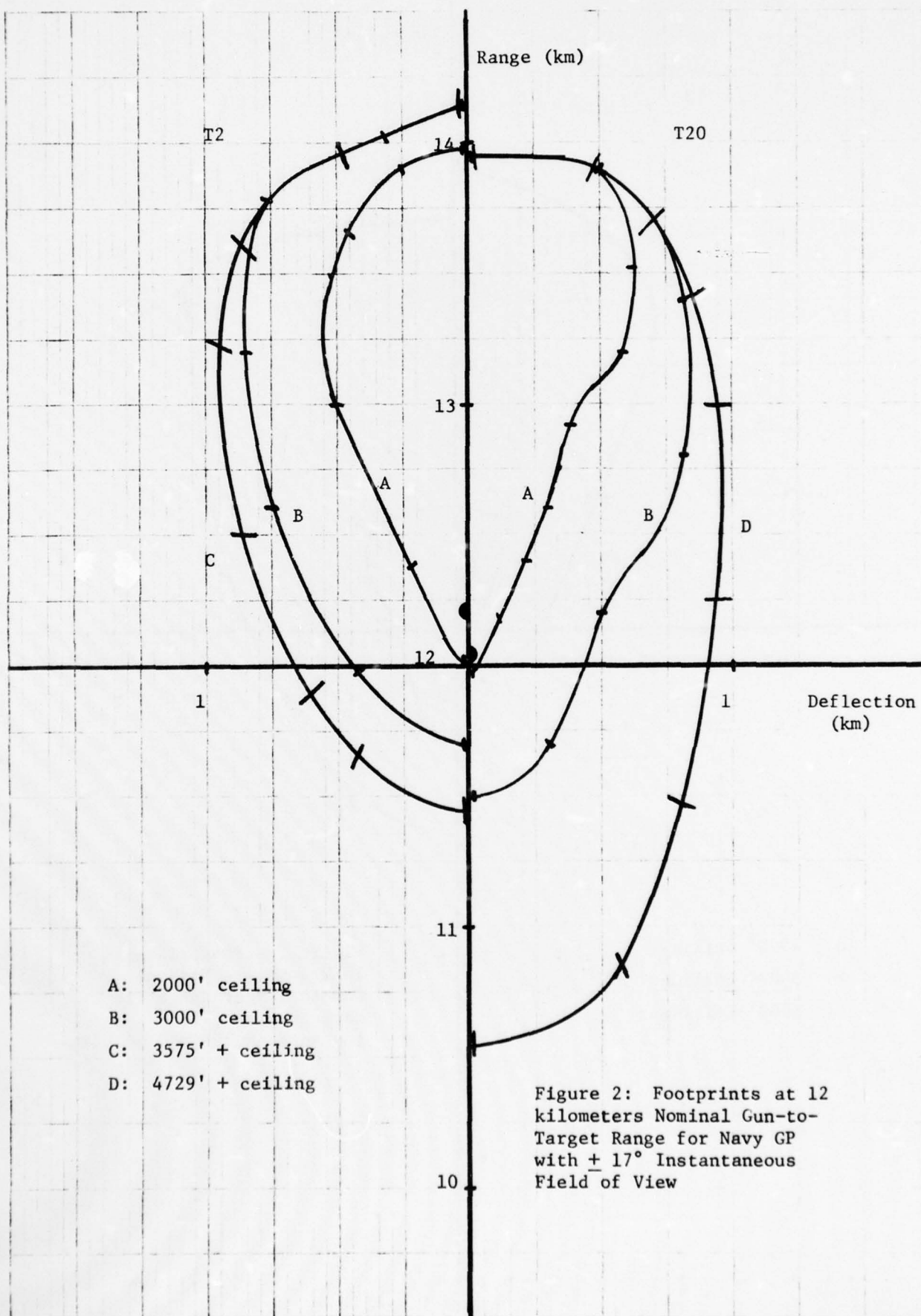
conclusions of Ref 1a in regard to selection of ignition delay option,  
nor does it change the conclusions of Ref 1b in regard to comparison of  
Copperhead vis-a-vis the Navy projectile.

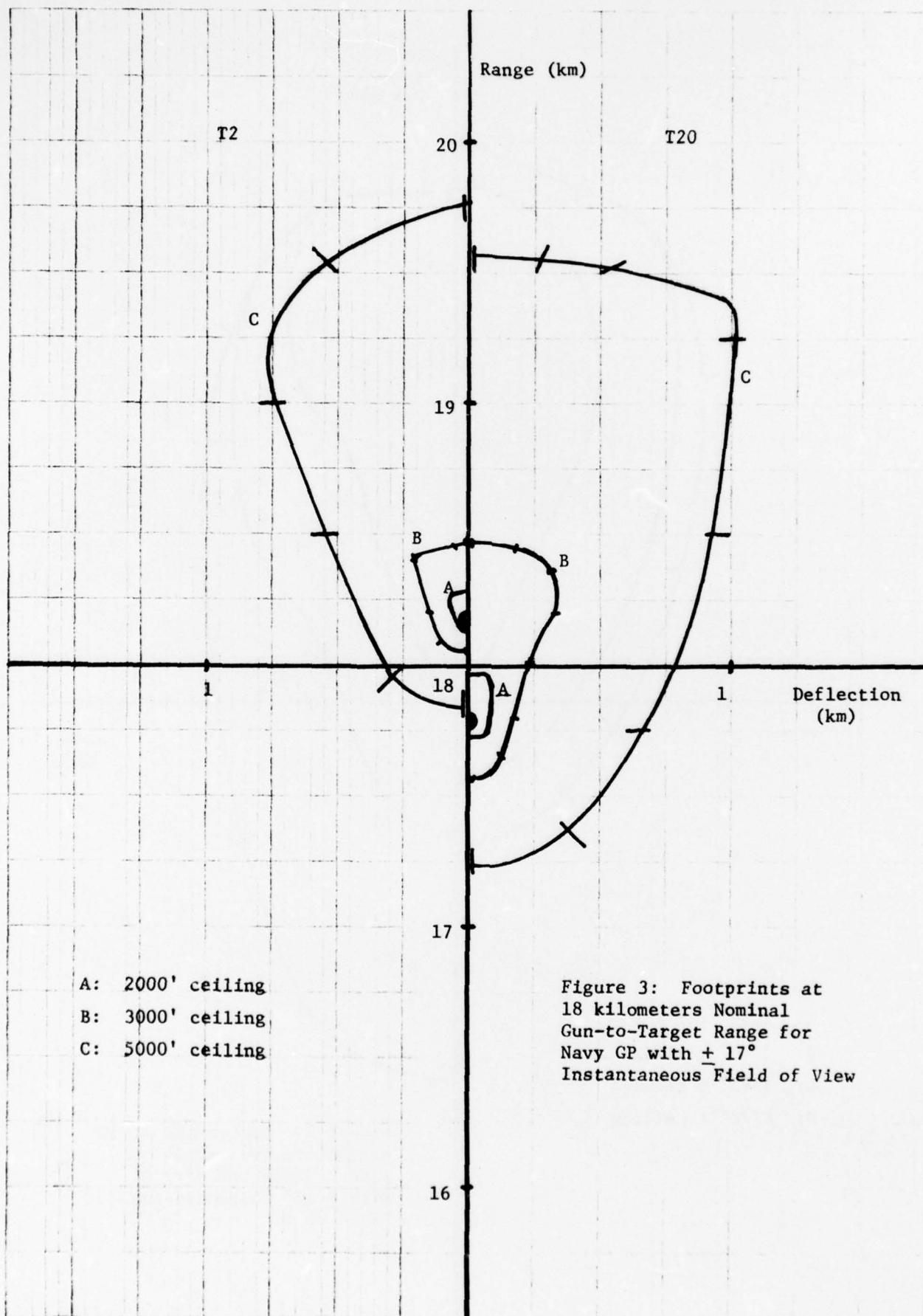


RICHARD D. HEIDER  
Operations Research Analyst  
Methodology Division  
Systems Analysis Directorate









A: 2000' ceiling  
 B: 3000' ceiling  
 C: 5000' ceiling

Figure 3: Footprints at  
 18 kilometers Nominal  
 Gun-to-Target Range for  
 Navy GP with  $\pm 17^\circ$   
 Instantaneous Field of View

TABLE 1: Footprint Areas for Navy GP with  $\pm 17^\circ$  FOV

Ceiling (ft)	6 km		12 km		18 km	
	T2	T20	T2	T20	T2	T20
948+	5.73	-----	-----	-----	-----	-----
2000	5.73	3.81	1.46	1.54	0.01	0.04
2336+	5.73	4.13	-----	-----	-----	-----
3000	5.73	4.13	3.05	3.12	0.13	0.41
3575+	5.73	4.13	3.95	-----	-----	-----
4729+	5.73	4.13	3.95	5.42	-----	-----
5000	5.73	4.13	3.95	5.42	2.15	3.70

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DRAFT INDEPENDENT EVALUATION PLAN (IEP)  
FOR DT I OF THE NEW PROTECTIVE MASK

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DRSAR-SA (2 Nov 76) 1st Ind  
SUBJECT: Draft Independent Evaluation Plan (IEP) for DT I of the New  
Protective Mask

HQ, US Army Armament Command, Rock Island, IL 61201


12 NOV 1976

TO: Commander, US Army Test and Evaluation Command, ATTN: DRSTE-SY,  
Aberdeen Proving Ground, MD 21005

DRSAR-SA has reviewed the subject draft IEP as requested. Our specific  
comments are provided as Incl 2 on DA Form 2028.

FOR THE COMMANDER:

1 Incl  
wd incl 1  
Added 1 incl  
2. DA 2028

  
MORRIS C. JOHNSON  
Acting Director  
Systems Analysis Directorate

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DEPARTMENT OF THE ARMY Mr. Ritondo/nr/AU 283-5279  
HEADQUARTERS, U. S. ARMY TEST AND EVALUATION COMMAND  
ABERDEEN PROVING GROUND, MARYLAND 21005

S-15 Nov 76

DRSTE-SY

2 NOV 1976

SUBJECT: Draft Independent Evaluation Plan (IEP) for  
DT I of the New Protective Mask

Commander  
US Army Armament Command  
ATTN: DRSAR-SA (Mr. O. Haase)  
Rock Island Arsenal, IL

1. Due to the tight schedule dictated by the accelerated schedule for development of the New Protective Mask, an abbreviated IEP has been drafted.
2. The attached DRAFT IEP for DT I of the New Protective Mask is forwarded for coordination with the ARMCOM Red Team. Your concurrence and/or comments are requested NLT 15 November 1976.
3. For additional information and/or questions relating to the IEP, contact Mr. Michael C. Ritondo, SAED, AUTOVON 283-5279/5280.

FOR THE COMMANDER:

RICHARD H. RIEL  
Director  
Sys Anal & Eval Directorate

1 Incl  
as

CF:  
Cdr, Edgewood Arsenal, ATTN: SAREA-DE-DPR (Mr. C. Shoemaker),  
Aberdeen Proving Ground, MD 21010

INCL 2



<b>RECOMMENDED CHANGES TO PUBLICATIONS AND BLANK FORMS</b>						Use Part II (reverse) for Repair Parts and Special Tool Lists (RPSTL) and Supply Catalogs/Supply Manuals (SC/SM).	DATE
For use of this form, see AR 310-1; the proponent agency is the US Army Adjutant General Center.							
TO: (Forward to proponent of publication or form) (Include ZIP Code) <b>Commander</b> <b>US Army Test &amp; Evaluation Command</b> <b>ATTN: DRSTE-SY</b> <b>Aberdeen Proving Ground, MD 21005</b>				FROM: (Activity and location) (Include ZIP Code) <b>Commander</b> <b>US Army Armament Command</b> <b>ATTN: DRSAR-SAM</b> <b>Rock Island, IL 61201</b>			
PART I - ALL PUBLICATIONS (EXCEPT RPSTL AND SC/SM) AND BLANK FORMS							
PUBLICATION/FORM NUMBER <b>Draft IEP</b>				DATE <b>2 Nov 76</b>		TITLE <b>DT I Independent Evaluation Plan for XM29 Protective Mask</b>	
ITEM NO.	PAGE NO.	PARA-GRAPH	LINE NO.*	FIGURE NO.	TABLE NO.	RECOMMENDED CHANGES AND REASON (Exact wording of recommended change must be given)	
1	2	B				COMMENT: This paragraph, as it exists, describes neither the threat nor the operational environment. It should be modified such that estimates of the expected dosage charge to masks can be made. Thus, it ties in with para E.1. on page 3.	
2	3	E.1.				COMMENT: The dosage level of $2 \times 10^4$ mg-min/m <sup>3</sup> seems to be excessive for any single massive attack. This dosage level should be reviewed with the thought of reducing it to a more realistic value.	
3	3	E.8.				CHANGE: The order of this subparagraph should be itemized among the critical issues. REASON: Paragraph B, page 2, points out that individuals may be required to assume CB protective postures for extended periods of time. Their ability to function in accomplishing their missions, while protected, is critical.	
4	4	E.11.				COMMENT: Must be modified to include some reasonable estimate of likely or maximum storage times. REASON: Recent unfortunate experience with the seal of the M17 mask after storage should form the basis for "lessons learned" with respect to degradation of chemical protective equipment in storage. Storage times should be related to specific types of problems which might accrue while the mask is in storage.	
5	4	E.14.				COMMENT: At least one other issue should be identified here: It has been noted that improperly fitted masks lead to leakages which can defeat the mask. Those improper fits can result from beards, dirt, and mismatches between mask sizes and individual head sizes. Proper	
*Reference to line numbers within the paragraph or subparagraph.							
TYPED NAME, GRADE OR TITLE <b>OTTO F. HAASE, JR.</b> <b>DRSAR-SAM</b>				TELEPHONE EXCHANGE/AUTOVON, PLUS EXTENSION <b>ATV 793-3177</b>		SIGNATURE	

**DA FORM 2028**  
1 FEB 74

REPLACES DA FORM 2028, 1 DEC 68, WHICH WILL BE USED.

*Encl 2*

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For use of this form, see AR 310-1; the proponent agency is the US Army Adjutant General Center.							
<b>TO:</b> (Forward to proponent of publication or form) (Include ZIP Code) Commander US Army Test & Evaluation Command ATTN: DRSTE-SY Aberdeen Proving Ground, MD 21005			<b>FROM:</b> (Activity and location) (Include ZIP Code) Commander US Army Armament Command ATTN: DRSAR-SAM Rock Island, IL 61201				
<b>PART I - ALL PUBLICATIONS (EXCEPT RPSTL AND SC/SM) AND BLANK FORMS</b>							
<b>PUBLICATION/FORM NUMBER</b>  Draft IEP				<b>DATE</b>  2 Nov 76		<b>TITLE</b> DT I Independent Evaluation Plan for XM29 Protective Mask	
ITEM NO.	PAGE NO.	PARA-GRAPH	LINE NO.*	FIGURE NO.	TABLE NO.	RECOMMENDED CHANGES AND REASON (Exact wording of recommended change must be given)	
6	3	E.7.				consideration of these factors in a test program could lead to corrective measures which would preclude casualties resulting from mask leakage.  <b>CHANGE:</b> Delete subparagraph E.7. <b>REASON:</b> The issue defined here is also defined under subparagraph D.4.	
7	4					<b>COMMENT:</b> Perhaps the question of compatibility of the mask with other military equipment should be identified as a critical issue and added to this subparagraph. Although equipment such as binoculars, crew serve weapons, etc. have been identified in the critical issues section, the matter of compatibility with individual weapons such as side arms, rifles, rocket launchers, communication gear, etc. have not been given adequate attention. Likewise, the compatibility of the mask with armor, aircraft, shelters, etc. has not been explored to the fullest extent possible.	
8	4	F				<b>COMMENT:</b> The statement is made here, without equivocation, that there are no known alternatives to the new protective masks. Surely the old protective masks, or product-improved versions of those old masks, should be considered alternatives.	
9	4	A				<b>COMMENT:</b> A methodology for evaluation of the storability of the masks will be required in order to determine the techniques for evaluation. Some indication of the likely failure modes and the consequences would be desirable.	
10	6	E.15.				<b>COMMENT:</b> Consistent with item 5 on page 1 of this review and item E.15. should be added here.	
*Reference to line numbers within the paragraph or subparagraph.							
<b>TYPED NAME, GRADE OR TITLE</b>  OTTO F. HAASE, JR. DRSAR-SAM				<b>TELEPHONE EXCHANGE/AUTOVON, PLUS EXTENSION</b>  ATV 793-3177		<b>SIGNATURE</b>  <div style="border: 1px solid black; height: 40px; width: 100%;"></div>	

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For use of this form, see AR 310-1; the proponent agency is the US Army Adjutant General Center.									
TO: (Forward to proponent of publication or form) (Include ZIP Code) Commander US Army Test & Evaluation Command ATTN: DRSTE-SY Aberdeen Proving Ground, MD 21005						FROM: (Activity and location) (Include ZIP Code) Commander US Army Armament Command ATTN: DRSAR-SAM Rock Island, IL			
<b>PART I - ALL PUBLICATIONS (EXCEPT RPSTL AND SC/SM) AND BLANK FORMS</b>									
PUBLICATION/FORM NUMBER Draft IEP						DATE 2 Nov 76		TITLE DT I Independent Evaluation Plan for XM29 Protective Mask	
ITEM NO.	PAGE NO.	PARA- GRAPH	LINE NO.	FIGURE NO.	TABLE NO.	RECOMMENDED CHANGES AND REASON (Exact wording of recommended change must be given)			
						Data requirements for information related to mask leakage problems exists.			
*Reference to line numbers within the paragraph or subparagraph.									
TYPED NAME, GRADE OR TITLE OTTO F. HAASE, JR. DRSAR-SAM						TELEPHONE EXCHANGE/AUTOVON, PLUS EXTENSION ATV 793-3177		SIGNATURE	

**DA FORM 2028**  
1 FEB 74

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MAXIMUM FEASIBLE (CRITICAL) DESIGNATION  
RANGE FOR COPPERHEAD AS A FUNCTION  
OF SEVERAL PARAMETERS

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DEPARTMENT OF THE ARMY  
HEADQUARTERS, UNITED STATES ARMY ARMAMENT COMMAND  
ROCK ISLAND, ILLINOIS 61201

REPLY TO  
ATTENTION OF:

DRSAR-SA

16 NOV 1976

SUBJECT: Maximum Feasible (Critical) Designation Range For Copperhead  
as a Function of Several Parameters

Commander  
Picatinny Arsenal  
ATTN: DRCPM-CAWS (Messrs. E. Manley & E. Zimpo)  
Dover, NJ 07801

1. References:

a. Meeting at MICOM with G&C Lab Personnel, 14 Oct 76, subject: Laser Designator Characteristics and Implications for Terminal Guidance.

b. Memorandum for Record, AMSAR-SAM, 7 Jan 76, subject: The Problem of Laser Pulses from the Surroundings During Target Tracking and Designation (Spillover).

c. Memorandum for Record, DRSAR-SAM, 11 Nov 76, subject: Proportion of Energy Spilled Over a Target During Tracking With a Laser Designator and Implications for Terminal Guidance.

2. During discussions at the referenced meeting (Ref 1a), plans were made to investigate the guidance accuracy of HELLFIRE and Copperhead when used with several different laser designators. The intent of the planned investigation is to provide an estimate of the maximum range, for each designator, for which the flight vehicles satisfy their guidance accuracy requirements.

3. Reference 1b treats this subject analytically, in a simplified manner, under the assumption that the maximum feasible designation range is determined by the occurrence of laser pulses from the background created by spillover. However, Ref 1b did not explicitly treat certain parameters of interest and assumed that the energy density across the laser beam was uniform.

4. In an effort to guide the selection of parameter values for more detailed (and costly) simulation-based studies, DRSAR-SA has elaborated the approach taken in Ref 1b. A memorandum of the updated study (Ref 1c)

DRSAR-SA

14 NOV 1975

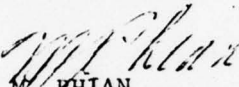
SUBJECT: Maximum Feasible (Critical) Designation Range For Copperhead  
as a Function of Several Parameters

is attached as Incl 1. In Ref 1c, the beam is given a gaussian energy profile and the Copperhead signal processing parameters associated with the target selection logic are treated explicitly. Additional parameters associated with the geometry of the scenario are also treated explicitly. Reference 1c also contains an expanded range of values of the parameters treated in Ref 1b.

5. Results in Ref 1c are provided for your information to update and supplement the preceeding information provided in Ref 1b.

FOR THE COMMANDER:

1 Incl  
as

  
M. KHIAN  
Acting Director  
Systems Analysis Directorate

CF:

Cdr, Picatinny Arsenal, ATTN: DRCPM-CAWS/Mr. B. Barrett, Dover, NJ 07801  
Cdr, USAVSCOM, P.O. Box 209, ATTN: DRSAR-WR/Mr. D. Dunlap, St. Louis, MO 63108  
Cdr, USAMICOM, Attn: ERSMI-RGT/Mr. C. Lewis, Huntsville, AL 35809  
Project Manager, Cannon Artillery Weapons Systems, Redstone Arsenal,  
ATTN: DRCPM-CAWS-FO/COL Nulk, Huntsville, AL 35809

DRSAR-SAM

: : NOV 1976

MEMORANDUM FOR RECORD

SUBJECT: Proportion of Energy Spilled Over a Target During Tracking With a Laser Designator and Implications for Terminal Guidance

1. References:

a. Letter, DRSAR-SAM, 18 Jul 76, subject: Request Review of Document-- Performance Requirements Tradeoff for Army Laser Designator Developments in Support of HELLFIRE and CLGP.

b. MFR, DRSAR-SAM, 17 Sep 76, subject: Review of the Response to GAO Information Request (No. 951283) Submitted by the HELLFIRE Project Office (HFPO).

c. MFR, AMSAR-SAM, 7 Jan 76, subject: The Problem of Laser Pulses from the Surroundings During Target Tracking and Designation (Spillover).

2. This memorandum is motivated in part by prior inquiries from the GAO and others into the relationship between laser designator characteristics and the guidance accuracy obtainable with associated laser guided systems such as Copperhead (CLGP) and HELLFIRE. This office has reviewed and supplemented information given the GAO by the HFPO on this subject (Refs 1a and 1b). Further motivation for this MFR is due to discussions held at MICOM on 14 Oct 76 relative to the above subject.

3. One can somewhat simplify the issue of guidance accuracy versus designator characteristics by restricting attention to assessing that range beyond which guidance accuracy fails to meet specified projectile performance requirements and/or beyond which there is a marked degradation of hit probability with additional designation range. In prior treatments of this subject such as Ref 1c, we have called this limiting range the maximum feasible (or critical) designation range. To determine the critical designation range with a given type of designator one must consider, inter alia, the question as to what is the maximum admissible amount of laser energy which is spilled over onto the background beyond the intended target per pulse. This limiting amount of spillover and the frequency with which it occurs is, of course, dependent upon a number of environmental (scenario-dependent) and projectile-dependent parameters. These are addressed in a simplified analytic treatment in Ref 1c.

4. In the following discussion additional attention will be given the distribution of laser energy over the beam cross section. (The energy

Incl 1

DRSAR-SAM

1 : MAY 1976

SUBJECT: Proportion of Energy Spilled Over a Target During Tracking With a Laser Designator and Implications for Terminal Guidance

density was taken as uniform in Ref 1c.) Further attention will be given the seeker processor logic for discriminating the target from the background, given spillover. In particular, the implementation of last-significant-pulse logic in Copperhead will be treated. Following Ref 1c, a simplified treatment of the target geometry will be used to estimate the frequency with which the seeker would track the background during terminal guidance. Background pulses which are tracked will be referred to as "significant."

5. Approximate Distribution of Energy in the Far-Field Beam Cross Section.

Altho macroscopic spacial variations in the atmospheric index of refraction ( $10^{-4}$ -10m) along the optical path of the laser produce stochastic variation in the distribution of energy across the beam at the target (as well as beam broadening and beam steering effects), the temporal mean distribution of energy across the beam in the far field is approximately gaussian (normal). In the analysis here the beam energy distribution is taken as circular normal. Consider the beam section shown in Figure 1. The radius of the beam,  $r_b$ , is defined as that value of  $r$  at which the inclosed circle contains 90% of the total energy.

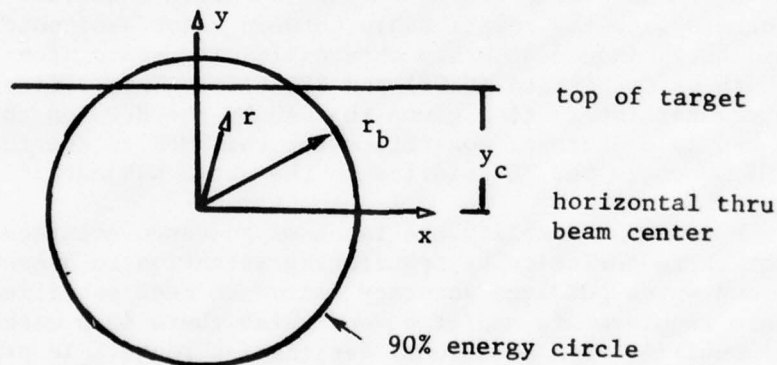


Figure 1. Beam Cross Section.

6. Let  $E$  represent the energy within a circle of radius  $r$ . Then, by assumption,

$$E = E_0 \left( 1 - \exp - \frac{1}{2} r^2 / \sigma_b^2 \right), \quad (1)$$



11 NOV 1976

DRSAR-SAM

SUBJECT: Proportion of Energy Spilled Over a Target During Tracking With  
a Laser Designator and Implications for Terminal Guidance

with total beam energy/pulse  $E_0$  and with

$$\sigma_b = r_b / \sqrt{-2 \ln(0.1)} \quad (2)$$

so that  $E(r_b) = 0.9$ .

From (1)

$$\frac{dE}{dr} = \frac{E_0}{2\sigma_b} r e^{-1/2(r/\sigma_b)^2} \quad (3)$$

7. Alternatively, the energy density with respect to each orthogonal coordinate is gaussian. For example,

$$d(E/E_0)/dy = \frac{1}{\sqrt{2\pi}\sigma_b} e^{-1/2(y/\sigma_b)^2} \quad (4)$$

Suppose that the vertical dimension of the target is critical, ie, much smaller than the horizontal extent of the target relative to the center of aim. We shall be concerned about the fraction of the total energy in the beam below the level  $y_c$  in Figure 1. This fraction,  $f$ , is obtained by integration of the density, given by (4).

$$f = \frac{1}{\sqrt{2\pi}\sigma_b} \int_{-\infty}^{y_c} e^{-1/2(y/\sigma_b)^2} dy$$

$$f = \Phi(y_c/\sigma_b) \quad , \text{ where } \Phi \text{ is the standard normal integral.} \quad (5)$$

As an approximation for  $\Phi$  for  $y_c/\sigma_b > 0$ ,

$$f = 0.5 + 0.5 [1 - \exp(-\frac{2}{\pi}(y_c/\sigma_b)^2)]^{1/2} \quad (6)$$

where the maximum error in this approximation over the interval--  
 $0 < y_c/\sigma_b < \infty$  -- is less than about 0.003.

8. From (2) and (6),

$$f = 0.5 + 0.5[1 - \exp(-2.9317 y_c^2/r_b^2)]^{1/2} \quad (7)$$

$$y_c > 0.$$



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Thus,  $f$  is the fraction of the incident energy which falls on the target when the beam center is at a position  $y_c$  below the top edge of the target. To obtain the relative energy which is reflected from target and background one must take note of the reflectance of target and background. Notationally, let  $\rho_t$  represent the reflectance of the target and  $\rho_b$  that of the background. Then, the ratio of reflected energies--background to target--is

$$\rho_b(1-f)/(\rho_t f) .$$

The energy from the background will travel a greater distance to reach the seeker than the energy from the target. In calculating the relative irradiance of the seeker dome contributed by these two sources, one must account for these different propagation paths. Thus, a relative attenuation of the background and target energies will be proportional to the square of the ratio of slant range from seeker to target,  $R_t$ , and from seeker to background,  $R_b$ . For the present purpose, effects due to differences in atmospheric extinction are ignored. An expression for  $R_b$  in terms of parameters of the engagement geometry is developed below. Notationally, let

$$\gamma = (R_t/R_b)^2 , \quad (8)$$

and

$$\rho = \rho_b/\rho_t . \quad (9)$$

Then, the ratio of the irradiance signals produced by background and target is

$$E_b/E_t = \gamma\rho(1-f)/f , \quad (10a)$$

and, from (7),

$$E_b/E_t = \frac{\gamma\rho\{0.5-0.5[1-\exp(-2.9317 y_c^2/r_b^2)]^{1/2}\}}{0.5+0.5[1-\exp(-2.9317 y_c^2/r_b^2)]^{1/2}} . \quad (10b)$$

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# 9. Target Selection Algorithm.

In processing the laser pulses reflected from target and background the seeker of a laser guided projectile implicitly uses a critical or threshold value for the ratio  $E_b/E_t$  to select a target to track. For values of  $E_b/E_t$  greater than this value the background will be tracked. Otherwise, the intended target will be tracked. To see how this happens, consider the situation shown in Figures 2a and b. The reflected energy at the seeker is depicted in Figure 2c. In Figure 2a the origin is directly below the projectile (P). The target is in the ground plane at T, and the background return emanates from B. The azimuthal difference between designator-to-target line and projectile-to-target line is  $\phi$ . The elevation of the projectile above the ground plane is  $\theta$ . The reflecting object in the background is considered sharply defined.

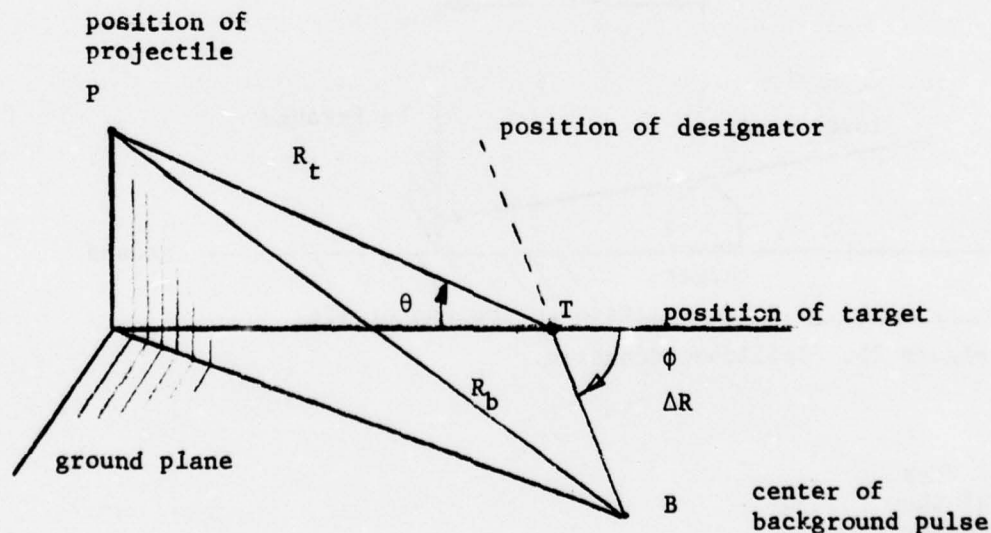


Figure 2a. Engagement Geometry.

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From the above geometry

$$R_b^2 = R_t^2 + \Delta R^2 + 2 R_t \Delta R \cos \theta \cos \phi.$$

Then,

$$\gamma = [1 + (\Delta R/R_t)^2 + 2 (\Delta R/R_t) \cos \theta \cos \phi]^{-1} \quad (11)$$

For example with these parameter values:  $R_t = 5000$  ft,  $\Delta R = 2000$  ft,  $\theta = 20$  deg,  $\phi = 25$  deg, the value of  $\gamma = 0.543$ . For Copperhead these values are considered representative.

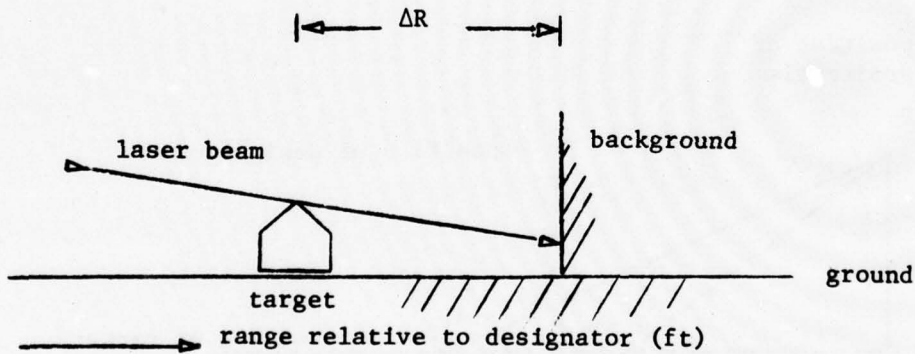


Figure 2b. Spillover Scenario.

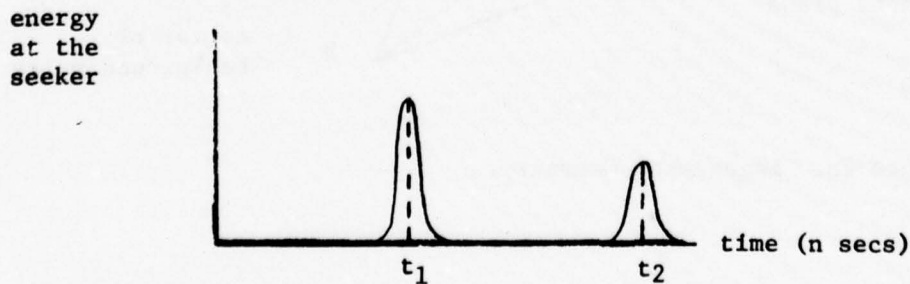


Figure 2c. Reflected Pulses at the Seeker.

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10. The energy at the seeker due to reflection from the target is indicated at time  $t_1$ . The energy return from the background occurs at  $t_2$ . For small azimuthal difference between designator and seeker\*, the time interval in nanosecs:  $t_2 - t_1$  is equal to the difference in path length in feet for the two paths--target to seeker and target to background to seeker. Exactly,

$$t_2 - t_1 = \Delta R + R_b - R_t \quad (12a)$$

or

$$t_2 - t_1 = \Delta R + R_t (\gamma^{-1/2} - 1). \quad (12b)$$

A useful approximation for our purpose is

$$t_2 - t_1 \text{ (n sec)} \approx 2 \Delta R \text{ (ft)}. \quad (13)$$

In the example given above the exact value of this time interval is 3785 n sec compared to 4000 n sec for the approximation.

11. The target selection algorithm employed in Copperhead establishes a dynamic threshold illustrated in Figure 3.

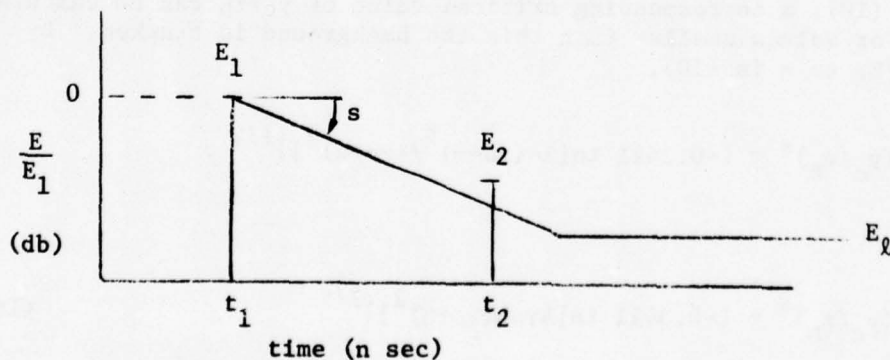


Figure 3. Plot of Energy Returns and Dynamic Threshold for Copperhead

\* Background energy centroid must remain in the seeker field of view for this analysis to hold.



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Assuming that both  $t_1$  and  $t_2$  lie within admissible time gates ( $50 \mu \text{ sec}$  max for  $t_1$ ), the occurrence of a target pulse at  $t_1$ , whose maximum amplitude is represented as  $E_1$ , generates a linearly decreasing threshold  $E/E_1$  on a logarithmic scale. The slope,  $s$ , of this threshold is presently  $0.5 \text{ db}/\mu \text{ sec}$ . This threshold decreases until a fixed relative minimum  $E_\ell$  occurs. For example,  $E_\ell$  is approximately 16 db below  $E_1$ . The selection logic is such that if  $E_2 > E(t_2)$ , the background is tracked. Otherwise, the target pulse is accepted and the target is tracked. The threshold value of  $E_2$ ,  $E_2^*$ , at which the background is tracked is given by

$$10 \log_{10} (E_2^*/E_1) = -s (t_2 - t_1), E_2 > E_\ell. \quad (14)$$

Approximately, from (13),

$$\eta \cong E_2^*/E_1 = \max (10^{-2 s \Delta R}, E_\ell/E_1). \quad (15)$$

Thus, with  $s$  and  $\Delta R$  fixed (in a particular scenario), the threshold ratio  $E_2^*/E_1$  is fixed. For example, with  $s = 5 \cdot 10^{-4} \text{ db}/\mu \text{ sec}$  and  $\Delta R = 2000 \text{ ft}$ ,

$$\eta = E_2^*/E_1 = 0.63.$$

12. In summary, the choice of parameters  $s$  and  $\Delta R$  fixes the minimum ratio,  $E_b/E_t$ , of reflected energies from background and target, for which the background will be tracked, generating a significant background pulse. By equation (10), a corresponding critical value of  $y_c/r_b$  can be calculated,  $(y_c/r_b)^*$ . For values smaller than this the background is tracked. By equating  $E_b/E_t$  to  $\eta$  in (10),

$$(y_c/r_b)^* = \{-0.3411 \ln[1 - (\gamma\rho - \eta)^2 / (\gamma\rho + \eta)^2]\}^{1/2}.$$

or

$$(y_c/r_b)^* = \{-0.3411 \ln[4\gamma\rho\eta / (\gamma\rho + \eta)^2]\}^{1/2}. \quad (16)$$



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13. The beam radius in the far field is given approximately in terms of the beam divergence,  $\beta$ , and designation range, R:

$$r_b = \beta R/2, \text{ with} \quad (17)$$

$\beta$  in (milliradians) and R in (km) for  $r_b$  in (m).

For a constant beam radius, the nearest the beam centroid can approach the target edge without a significant background pulse is given by

$$y_c^* = (y_c/r_b)^* \beta R/2. \quad (18)$$

14. The Critical Designation Range.

Using the notation in Ref 1c, let the distance from the center of aim\* on the target to the top edge of the target be L. Then, the critical value (level) of the beam centroid relative to the center of aim\* for significant background pulses is

$$u = L - y_c^*. \quad (19)$$

If the beam centroid makes an excursion greater than u, a significant background pulse will occur and be repeated as long as u is exceeded.

15. Following the practice of Ref 1c, the critical designation range is defined as that which produces a statistical expectation of three significant sequences of background pulses during a typical ten-second guidance period. The expected number of significant sequences is identical to the expected number of crossings of the level u in time T. From Ref 1c, the expected level crossings of a stationary stochastic process having second-order Butterworth dynamics and a one hertz corner frequency is

$$E(C) = 2T \exp(-u^2/2\sigma_c^2), \quad ** \quad (20)$$

---

\* The center of aim is identified with the mean vertical coordinate of the spot motion occurring during tracking by the designator. Tracking jitter is treated as a stationary stochastic process.

\*\* This result is derived in Annex 1.

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where  $\sigma_t$  is the standard deviation of the tracking error. Over the designation ranges of interest  $\sigma_t$  is a linear function of range. Thus,

$$\sigma_t = s_t R, \quad (21)$$

with  $s_t$  in (milliradians) and  $R$  in (km) to yield  $\sigma_t$  in (m).

From (20),

$$E(C) = 2T \exp[1 - u^2 / (2 s_t^2 R^2)]. \quad (22)$$

16. Applying the criterion for critical designation range  $R^*$ , we set  $E(C) = 3$  and  $T = 10$  sec. Then, solving for  $u(R^*)/R^*$  in (22):

$$u(R^*)/R^* = s_t \sqrt{-2 \ln(3/20)} = 1.948 s_t. \quad (23)$$

From (18, 19, 23), the critical designation range in (km) is

$$R^* = L / [1.948 s_t + 0.5 (y_c/r_b)^* \beta]. \quad (24)$$

The value of  $(y_c/r_b)^*$  is given by (16).

#### 17. An Example.

Take the computed value of  $\eta = 0.63$  from the previous example as well as the following parameters:  $\rho = 7$ ,  $\gamma = 0.54$ ,  $L = 0.8$  (m),  $\beta = 0.15$  (millirad), and  $s_t = 0.1$  (millirad). For these values  $R^* = 3.45$  km. A comparable result given in Ref 1c for the same parameters but having a uniformly distributed beam energy produces  $R^*$  equal to 3.30 km, to three significant figures. Other parametric results of the above theory are given in Figures 4 thru 6 and Tables 1 thru 5.

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18. Interpretation of Results.

Among the parameters examined several can be disposed of as being of slight importance in determining the critical designation range,  $R_{crit}$ . For this group the critical range is not very sensitive to changes in parameter values over the range of values examined (and expected operationally). For example, target-to-background range and seeker-to-target slant range are not particularly sensitive. Neither is the relative azimuth between the seeker-to-target and designator-to-target lines (seeker-designator azimuth). See Tables 1, 2, and 3. The background/target reflectance ratio,  $\rho$ , is somewhat more sensitive, producing a change of about 13% in  $R_{crit}$  while varying over the decade from 1 to 10. The reason for the greater sensitivity of  $\rho$  is illustrated in Figure 4. The critical position of the beam centroid with respect to the edge of the target is shown as a function of  $\rho$ . This critical position is seen to change from near zero to more than one-half a beam radius as  $\rho$  varies over the decade from 1 to 10. For most natural backgrounds and for Soviet target vehicles, the expected reflectance ratio is 7. This is the constant value of  $\rho$  chosen when other parameters are varied. In the same spirit, the seeker-designator azimuth was fixed at 25 deg when other factors were varied parametrically. The median attack azimuth obtained in operational combat simulations with Copperhead was 25 degrees. Similarly, the distance from target to background ( $\Delta R$ ) expected when significant laser spillover occurs during designation from well sited surface positions lies in the range from 1000 to 5000 ft, with 2000 ft being a typical value. Therefore, this constant value of  $\Delta R$  was used for other parametric analyses.

19. One of the seeker parameters considered significant to target selection is the slope,  $s$ , of the seeker (target discrimination) threshold. The nominal value of  $s$  for Copperhead is 0.5 db/ $\mu$  sec. However, changes in  $s$  by a factor of 2 (or 1/2) produce only slight changes in the critical position of the beam centroid (Table 5) and even smaller changes in the critical designation range (Table 3).

20. The most sensitive parameters affecting  $R_{crit}$  are beam divergence and tracking error. Of these parameters tracking error is the more significant, particularly for small values of beam divergence. See Table 4 and Figures 5 and 6. As long as beam divergence remains below about 0.2 milliradians the dominating effect of tracking accuracy is evident. To be sure, small improvements in  $R_{crit}$  can be obtained by having small (and fortunate) values of  $\rho$  or by making optimal adjustments in the seeker signal processing parameters or by further reduction in beam divergence, but the greatest improvement must depend

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upon reduction of the standard deviation (SD) of spot motion. For example, halving the SD from 0.10 mr to 0.05 mr increases  $R_{crit}$  from 3.3 to 5.5 km, with a beam divergence of 0.2 mr and other parameters fixed at nominal. Since this increased designation range has some operational utility for Copperhead, the effort to reduce tracking error should be pursued.

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TABLE 1. EFFECTS OF TARGET-BACKGROUND  
DISTANCE AND REFLECTANCE RATIO  
ON THE CRITICAL DESIGNATION RANGE

Table entries are critical ranges in (km).

Target-to Background (ft)	Background/Target Reflectance Ratio				
	1	3	5	7	9
1000	4.069	3.832	3.711	3.639	3.590
2000	4.047	3.851	3.727	3.654	3.604
3000	4.036	3.860	3.736	3.662	3.611
4000	4.034	3.862	3.737	3.663	3.612
5000	4.040	3.857	3.733	3.659	3.608

Constant Parameters

seeker-target slant range	5000 ft
seeker-designator azimuth	0 deg
seeker elevation	20 deg
seeker threshold slope	0.5 db/ $\mu$ sec
designator beam div	0.10 m r
designator tracking error SD	0.10 m r



TABLE 2. EFFECTS OF SEEKER-TO-DESIGNATOR  
AZIMUTH AND REFLECTANCE RATIO  
ON THE CRITICAL DESIGNATION RANGE

Table entries are critical ranges in (km).

Seeker-to- Designator Azimuth (deg)	Background/Target Reflectance Ratio				
	1	3	5	7	9
0	4.047	3.851	3.727	3.654	3.604
15	4.021	3.849	3.725	3.652	3.602
25	4.054	3.845	3.722	3.650	3.599
35	4.060	3.840	3.717	3.645	3.595
45	4.070	3.832	3.710	3.639	3.589

Constant Parameters

seeker-target slant range	5000 ft
target-background distance	2000 ft
seeker elevation	20 deg
seeker threshold slope	0.5 db/ $\mu$ sec
designator beam divergence	0.10 mr
designator tracking error SD	0.10 mr

TABLE 3. EFFECTS OF SEEKER-TO-TARGET  
SLANT RANGE AND SEEKER THRESHOLD  
SLOPE ON THE CRITICAL DESIGNATION RANGE

Table entries are critical ranges in (km).

Seeker Azimuth (deg)	Seeker Slant Range (ft)	Seeker (Target Discrimination) Threshold Slope (db/ $\mu$ sec)		
		0.25	0.50	1.00
25	5000	3.696	3.650	3.565
0	5000	3.702	3.654	3.566
0	2000	3.873	3.815	3.708
0	1000	4.106	4.037	3.908

Constant Parameters

target-background distance	2000 ft
seeker elevation	20 deg
background/target reflectance	7
designator beam divergence	0.10 mr
designator tracking error SD	0.10 mr

TABLE 4. EFFECTS OF DESIGNATOR BEAM  
DIVERGENCE AND TRACKING ACCURACY ON  
THE CRITICAL DESIGNATION RANGE

Table entries are critical ranges in (km).

Beam Divergence (milliradians)	Background/Target Reflectance = 3				
	Tracking Error Standard Deviation (milliradians)				
	0.05	0.10	0.15	0.20	0.25
0.10	7.230	3.845	2.619	1.986	1.599
0.15	6.821	3.726	2.563	1.954	1.578
0.20	6.456	3.615	2.510	1.923	1.558
0.25	6.128	3.510	2.459	1.892	1.538
0.30	5.832	3.410	2.410	1.863	1.519

Background/Target Reflectance = 7					
0.10	6.568	3.650	2.527	1.932	1.564
0.15	5.969	3.457	2.433	1.877	1.528
0.20	5.471	3.284	2.346	1.825	1.493
0.25	5.049	3.127	2.265	1.775	1.460
0.30	4.688	2.985	2.189	1.728	1.428

Constant Parameters

seeker-target slant range	5000 ft
target-background distance	2000 ft
seeker-designator azimuth	25 deg
seeker elevation	20 deg
seeker threshold slope	0.5 db/ $\mu$ sec

TABLE 5. CRITICAL POSITION OF BEAM CENTROID  
 $(y_c/r_b)^*$  AS A FUNCTION OF REFLECTANCE  
 RATIO ( $\rho$ ) AND SEEKER THRESHOLD SLOPE ( $s$ )

Reflectance Ratio, $\rho$	Seeker Threshold Slope, $s$ (db/ $\mu$ sec)		
	0.25	0.50	1.00
1	.0626	.0311	.0319
2	.1391	.1700	.2313
3	.2541	.2840	.3428
4	.3330	.3620	.4186
5	.3924	.4204	.4753
6	.4395	.4669	.5201
7	.4784	.5051	.5571
8	.5114	.5375	.5883
9	.5398	.5654	.6153
10	.5648	.5900	.6390

Constant Parameters

seeker-target slant range	5000 ft
target-background distance	1000 ft
seeker-designator azimuth	25 deg
seeker elevation	20 deg



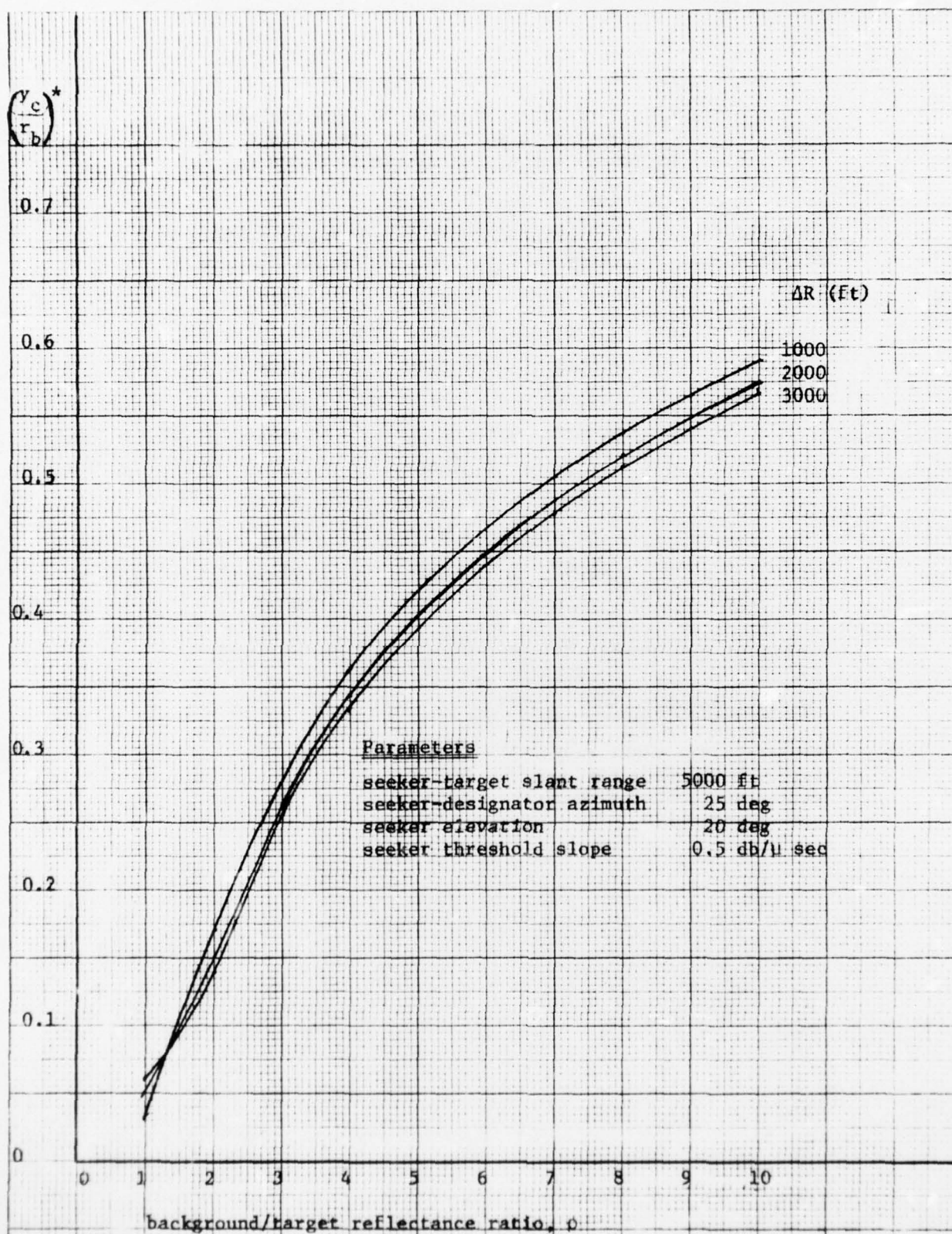
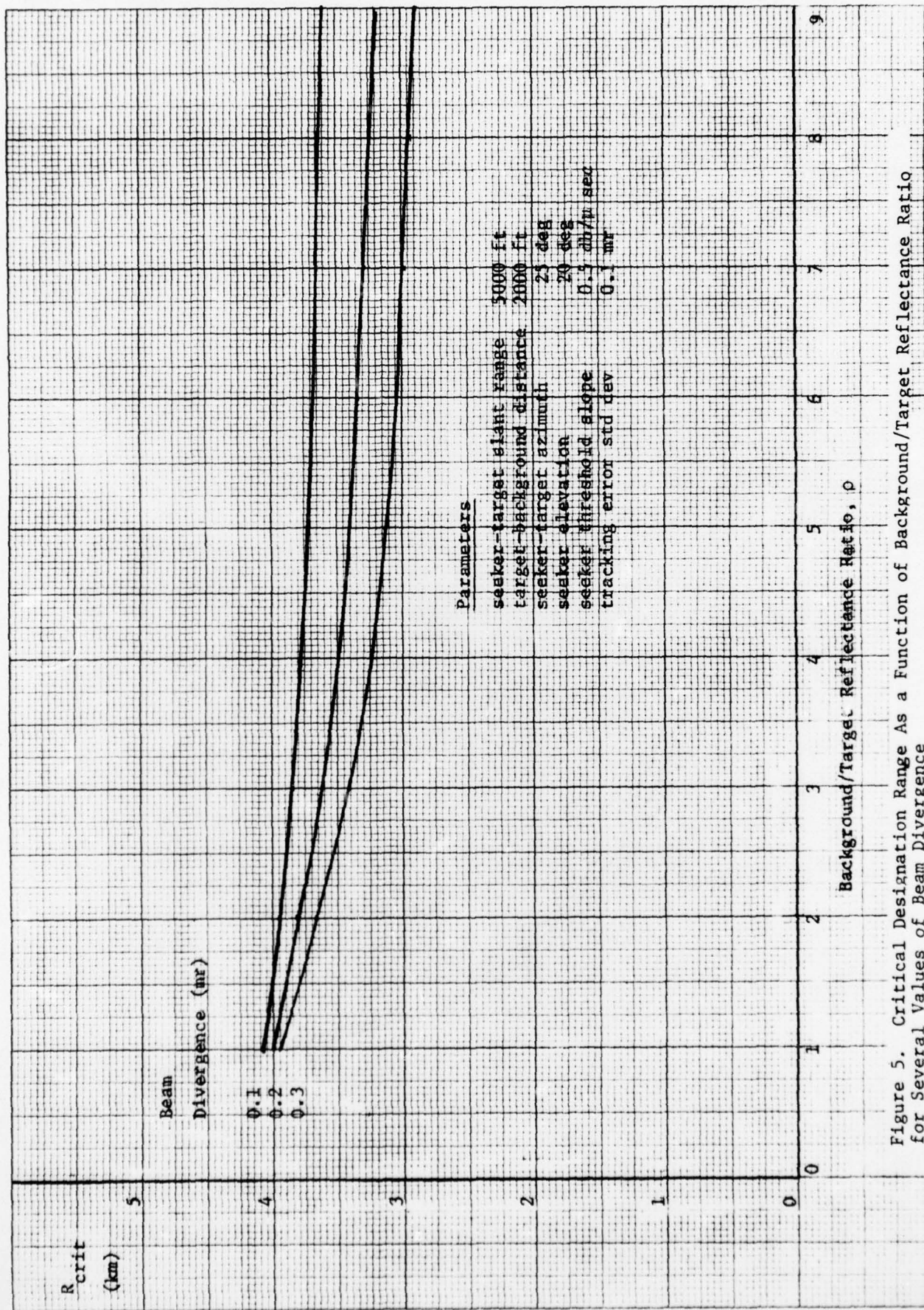


Figure 4. Critical Position of Beam Centroid  $(y_c/r_b)^*$  as a Function of Reflectance Ratio ( $\rho$ ) and Target-Background Distance ( $\Delta R$ )





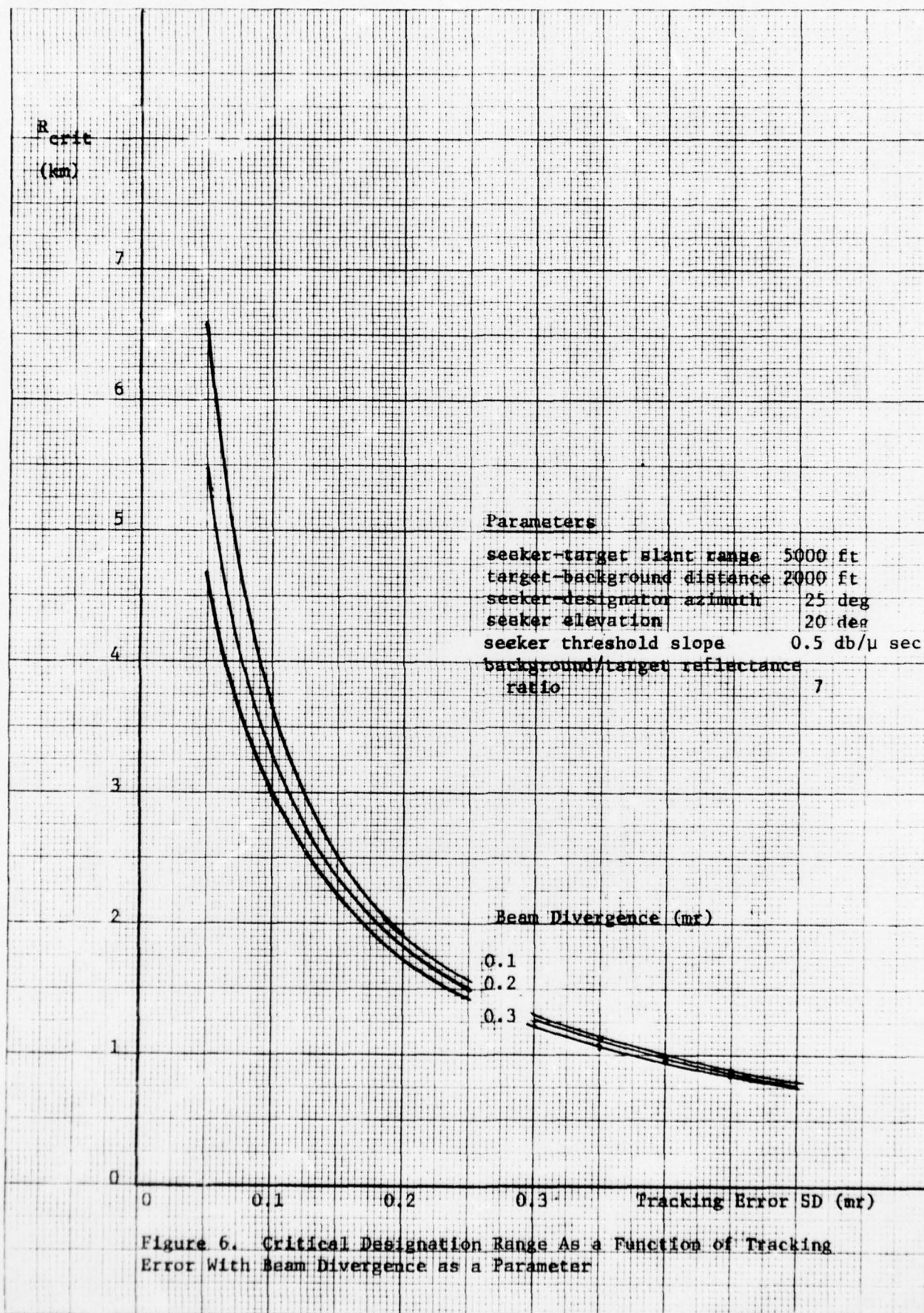


Figure 6. Critical Designation Range As a Function of Tracking Error With Beam Divergence as a Parameter

# ANNEX 1

## A Result on Level-Crossings for Second-Order Stochastic Processes

From p. 437, "Stochastic Point Processes: Statistical Analysis, Theory and Applications," Lewis, 1972:\*

The mean number of crossings of a level  $u$  by a stationary (zero mean) normal stochastic process  $\xi(t)$  in an interval  $0 \leq t \leq T$

$$E(C) = \frac{T}{\pi} \left( \frac{\lambda_2}{\lambda_0} \right)^{1/2} \exp(-u^2/2\lambda_0) \quad , \quad (1a)$$

where, if  $r(\tau)$  denotes the covariance function of  $\xi(t)$ ,

$$\lambda_0 = r(0) = \text{var } \xi(t) \quad (1b)$$

and  $\lambda_2 = -r''(0)$ , the second derivative of  $r$  at the origin. Note that  $\lambda_2$  is also the second spectral moment

$$\lambda_2 = \int_0^\infty \lambda^2 dF(\lambda) \quad (1c)$$

if  $F$  denotes the spectral function for  $\xi(t)$ .

### An Example

The transfer function for a second-order Butterworth stochastic system is

$$H_x(s) = \frac{K \omega_a^2}{s^2 + \sqrt{2} \omega_a s + \omega_a^2} \quad (2)$$

where  $K$  is the amplitude constant and

$$\omega_a = 2 \pi \nu_a$$

\*Lewis, P. A. Stochastic Point Processes: Statistical Analysis, Theory and Applications, Wiley Interscience, New York, c. 1972.



with  $\nu_a$  the analog corner frequency in hertz.

The transfer function produces the spectral density as follows:

$$\begin{aligned}\Gamma_{XX}(\omega) &= |H_X(j\omega)|^2 \\ &= \frac{K^2 \omega_a^4}{[(j\omega)^2 + \sqrt{2} j \omega_a \omega + \omega_a^2] [-\omega^2 - \sqrt{2} j \omega_a \omega + \omega_a^2]} \\ &= \frac{K^2 \omega_a^4}{(\omega_a^2 - \omega^2)^2 + 2 \omega_a^2 \omega^2} \\ &= \frac{K^2 \omega_a^4}{\omega_a^4 + \omega^4}\end{aligned}$$

$$\Gamma_{XX}(\omega) = \frac{K^2}{1 + (\omega/\omega_a)^4} \quad (3)$$

$$\Gamma'_{XX}(f) = \frac{K^2}{1 + f^4} \quad (4)$$

where

$$f = \omega/\omega_a$$

If we require that the process have unity variance,

$$\int_0^\infty \Gamma_{XX}(\omega) d\omega = 1 \quad (5)$$

and

$$\int_0^\infty \frac{K^2 d\omega}{1 + (\omega/\omega_a)^4} = 1$$

$$\omega_a K^2 \int_0^{\infty} \frac{d(\omega/\omega_a)}{1 + (\omega/\omega_a)^4} = 1$$

$$K^2 = \omega_a^{-1} / \int_0^{\infty} \frac{df}{1 + f^4} \quad (6)$$

But with

$$\int_0^{\infty} \frac{x^{p-1} dx}{1 + x^q} = \frac{\pi}{q \sin(\frac{p\pi}{q})} \quad (7)$$

$$K^2 = \omega_a^{-1} / \left( \frac{\pi \sqrt{2}}{4} \right)$$

$$K^2 = \frac{4}{\omega_a \pi \sqrt{2}} \quad (8)$$

The second spectral moment for a second-order Butterworth noise process is

$$\begin{aligned} \lambda_2 &= \int_0^{\infty} \omega^2 \Gamma_{xx}(\omega) d\omega \\ &= K^2 \omega_a^3 \int_0^{\infty} \frac{f^2 df}{1 + f^4} \\ \lambda_2 &= \frac{4 \omega_a^2}{\pi \sqrt{2}} \int_0^{\infty} \frac{f^2 df}{1 + f^4} \quad (9) \end{aligned}$$

Then, with (7)

$$\lambda_2 = \frac{4 \omega_a^2}{\pi \sqrt{2}} \frac{\pi}{4 \sin(\frac{3\pi}{4})} = \omega_a^2 \quad (10)$$

From (1), the expected number of crossings of level  $u$  in time  $T$  for a second-order Butterworth process with variance  $\sigma^2$  assumes the simple form

$$E(C) = \frac{T \omega_a}{\pi} \exp(-u^2/2\sigma^2)$$

or

$$E(C) = 2 T \nu_a \exp(-u^2/2\sigma^2) \quad (11)$$

where  $\nu_a$  is the corner frequency.

